Question			Answer	Marks	Guidance
1	(i)		$mv = (m+\delta m)(v+\delta v) + (-\delta m)(v-u)$ (note $\delta m < 0$)	M1 A1	Attempt at momentum equation Condone wrong sign of δm
			$mv = mv + v\delta m + m\delta v + \delta m\delta v - v\delta m + u\delta m$		
			$m\dfrac{\delta v}{\delta t} + u\dfrac{\delta m}{\delta t} + \delta m\dfrac{\delta v}{\delta t} = 0$	M1	Simplify and divide by δt
			$m\dfrac{\mathrm{d}v}{\mathrm{d}t} = -u\dfrac{\mathrm{d}m}{\mathrm{d}t}$		
			$\dfrac{\mathrm{d}m}{\mathrm{d}t} = -k$	B1	
			$m = m_0 - kt$	B1	SOI
			$(m_0 - kt)\dfrac{\mathrm{d}v}{\mathrm{d}t} = uk$	E1	All correct including sign of δm
				[6]	
1	(ii)		$\int \mathrm{d}v = \int \dfrac{uk}{m_0 - kt}\mathrm{d}t$	M1	Separate and integrate
			$v = -u\ln(m_0 - kt) + c$	A1	
			$t = 0, v = v_0 \Rightarrow v_0 = -u\ln m_0 + c$	M1	Use condition
			$c = v_0 + u\ln m_0$	A1	
			$v = v_0 + u\ln\left(\dfrac{m_0}{m_0 - kt}\right)$	A1	aef
				[5]	
2	(i)		Let equilibrium extension be e		
			$mv\dfrac{\mathrm{d}v}{\mathrm{d}x} = mg - k(e+x)$	M1	N2L
				A1	All terms correct
			At equilibrium, $mg = ke$	B1	oe
			So $mv\dfrac{\mathrm{d}v}{\mathrm{d}x} = -kx$	E1	With evidence of working
				[4]	

Question			Answer	Marks	Guidance
2	(ii)		$\int mv\,\mathrm{d}v = \int -kx\,\mathrm{d}x$	M1	Solutions from SHM acceptable
			$\frac{1}{2}mv^2 = -\frac{1}{2}kx^2 + c$	A1	
			$x = a, v = 0 \Rightarrow 0 = -\frac{1}{2}ka^2 + c$	M1	
			$\frac{1}{2}mv^2 = \frac{1}{2}k\left(a^2 - x^2\right)$	A1	oe
			$v = -\sqrt{\frac{k}{m}\left(a^2 - x^2\right)}$	E1	AG Complete argument including justification for square root.
			($v < 0$ when moving up)	**[5]**	
2	(iii)		$\int \frac{1}{\sqrt{a^2 - x^2}}\,\mathrm{d}x = \int -\sqrt{\frac{k}{m}}\,\mathrm{d}t$	M1*	Solutions from SHM *NOT* acceptable
			$\arcsin\left(\frac{x}{a}\right) = -\sqrt{\frac{k}{m}}t + c_2$	A1	
			$x = a, t = 0 \Rightarrow \frac{1}{2}\pi = c_2$	DM1	Accept c_2 = arcsin 1
			$x = a\sin\left(\frac{1}{2}\pi - \sqrt{\frac{k}{m}}t\right) = a\cos\left(\sqrt{\frac{k}{m}}t\right)$	A1	Either form
				[4]	
3	(i)		$l = 2a\sin\theta$	M1	OE eg $a\sqrt{2 - 2\cos 2\theta}$
			$V = \frac{\lambda}{2a}(2a\sin\theta - a)^2$	A1	EPE OE eg $\frac{\lambda}{2a}(a\sqrt{2-2\cos 2\theta} - a)^2$
			$\ldots + mga\cos 2\theta$	M1	Both terms
				A1	GPE OE eg $mga\sin(\frac{1}{2}\pi - 2\theta)$
			$\frac{\mathrm{d}V}{\mathrm{d}\theta} = -2mga\sin 2\theta + \frac{\lambda}{a}\left(2a\sin\theta - a\right)\cdot 2a\cos\theta$	M1	Differentiate
			$= -4mga\sin\theta\cos\theta + 2\lambda a\cos\theta\left(2\sin\theta - 1\right)$	M1	Use trigonometric identities as necessary
			$= 2a\cos\theta(2\lambda\sin\theta - 2mg\sin\theta - \lambda)$	E1	
				[7]	

Question			Answer	Marks	Guidance
3	(ii)		$\theta=\frac{1}{2}\pi \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}\theta}=0\times(\ldots)=0$	M1	
			hence equilibrium	E1	
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}=-2a\sin\theta(2\lambda\sin\theta-2mg\sin\theta-\lambda)$ $+2a\cos\theta(2\lambda\cos\theta-2mg\cos\theta)$	M1 A1	Here or in (iii) or use sign method
			$\theta=\frac{1}{2}\pi \Rightarrow \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}=-2a(2\lambda-2mg-\lambda)$	M1	Use V'' or equivalent method
			So $\lambda<2mg \Rightarrow \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}>0 \Rightarrow$ stable	E1	
			If $\cos\theta\neq 0$		
			$\frac{\mathrm{d}V}{\mathrm{d}\theta}=0 \Leftrightarrow 2\lambda\sin\theta-2mg\sin\theta-\lambda=0$		
			$\Leftrightarrow \sin\theta=\frac{\lambda}{2\lambda-2mg}$	M1	Consider other solutions
			But $\lambda<2mg \Rightarrow 2\lambda-2mg<\lambda$		
			$\Rightarrow \sin\theta>1$ or $\sin\theta<0$	M1	Attempt at showing not valid
			So no valid solutions	E1	Must consider both ends
				[9]	
3	(iii)		If $\lambda>2mg, \theta=\frac{1}{2}\pi$ as before	B1	
			$V''<0$ so unstable	B1	
			or $\sin\theta=\frac{\lambda}{2\lambda-2mg}$	E1	
			and $\frac{1}{2}<\frac{\lambda}{2\lambda-2mg}<1$ so gives valid solution	E1	
			$\theta=\sin^{-1}\left(\frac{\lambda}{2\lambda-2mg}\right)$ or $\pi-\sin^{-1}\left(\frac{\lambda}{2\lambda-2mg}\right)$	B1	For both
			and $V''=0+2a\cos^2\theta(2\lambda-2mg)$	M1	
			$=(+\text{ve})(+\text{ve})$ so stable (in both cases)	A1	
				[7]	

Question			Answer	Marks	Guidance
4	(i)		$\delta I = 2\pi r \delta r \rho r^2$	B1	
			$\rho = \dfrac{m}{\pi a^2}$	B1	
			$I_{\text{disc}} = \int_0^a \dfrac{m}{2a^2} r^3 \, \mathrm{d}r$	M1	for $k\int r^3 dr$
			$= \dfrac{m}{2a^2}\left[\dfrac{1}{4}r^4\right]_0^a$	M1	$k\left[\frac{1}{4}r^4\right]_0^a$ with limits
				A1	$\frac{k}{4}a^4$
			$= \dfrac{1}{2}ma^2$	E1	
				[6]	
4	(ii)		$I = m_1 a^2$	M1	Curved surface $2\pi h\rho a^3$
			$+ \frac{1}{2}ma^2 \times 2$	M1	Combine $+ \frac{1}{2}\rho\pi a^4 \times 2$
			$m = M\dfrac{\pi a^2}{2\pi a^2 + 2\pi ah}$	B1	$m = \pi\rho a^2$
			$m_1 = M\dfrac{2\pi ah}{2\pi a^2 + 2\pi ah}$	B1	$m_1 = 2\pi ah\rho$
			So $I = Ma^2\left(\dfrac{\pi a^2 + 2\pi ah}{2\pi a^2 + 2\pi ah}\right)$	M1	Substitute $I = M\dfrac{\pi\rho a^4 + 2\pi\rho a^3}{2\pi\rho a^2 + 2\pi\rho ah}$
			$I = \dfrac{1}{2}Ma^2\left(\dfrac{a+2h}{a+h}\right)$	E1	
				[6]	
4	(iii)		$I = \dfrac{1}{2}\times 8 \times 0.5^2\left(\dfrac{0.5+0.6}{0.5+0.3}\right) = 1.375$	B1	
			$I(\omega - 0) = Ja$	M1	Impulse/momentum
			$1.375\omega = 55 \times 0.5$		
			$\omega = 20 \text{ rad s}^{-1}$	A1	
				[3]	

Question			Answer	Marks	Guidance
4	(iv)		$I\dfrac{\mathrm{d}\dot{\theta}}{\mathrm{d}t}=-2\dot{\theta}^2$	B1	
			$\int 1.375\dot{\theta}^{-2}\,\mathrm{d}\dot{\theta}=\int -2\,\mathrm{d}t$	M1	Separate
			$-\dfrac{1.375}{\dot{\theta}}=-2t+c$	M1 A1	Integrate
			$t=0,\dot{\theta}=20\Rightarrow c=-0.06875$	M1	Use condition
			$t=5\Rightarrow -\dfrac{1.375}{\dot{\theta}}=-10-0.06875$	M1	
			$\Rightarrow \dot{\theta}=0.137$ (3sf)	A1 **[7]**	
4	(v)		$I\left(\dfrac{-0.137}{t}\right)=-0.03$	M1 A1	Complete method with correct acceleration (or both sides +ve)
			$t=6.26$ s	A1 **[3]**	awfw [6.25, 6.3] CAO